**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: We have a normal distribution with = 45 and =8.0. Let X be the amount of time it takes to complete the repair on a customer’s car. To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50). Pr(X >50) = 1 - Pr(X ≤ 50). Z = (X - )/ =(X - 45)/8.0. Thus the question can be answered by using the normal table to find Pr(X ≤ 50)

= Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625) = 73.4%. Probablilty that the service manager will not meet his demand will be = 100 - 73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
4. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: *The difference between 2 X₁ and X₁ + X₂ is N (0, 6 σ²).*

**Step-by-Step explanation:**

According to the **Central Limit Theorem** , any **Large Sum of Independent Identically Distributed (iid)** random variable is approximately **Normal.**

The **Normal Distribution** is defined by two parameters, the **Mean** , µ , and the **Variance** , σ² and written as X ~ N (µ, σ²) .

Given X₁ ~ N (µ , σ²) and X₂ ~ N (µ , σ²) are two independent identically distributed random variables.

From the properties of **Normal Random Variables,**

if X ~ N(µ₁ , σ₁²) and Y ~ N(µ₂ , σ₂²) are two independent identically distributed random variables then.

**\*** The **Sum** of normal variables is given by

X + Y ~ N(µ₁ + µ₂ , σ₁²+ σ₂²),

\* and the **difference** of normal random variables is given by

X - Y ~ N(µ₁ - µ₂ , σ₁²+ σ₂²),

**\*** When Z = aX , the **Product** of X is given by

Z ~ N (aµ₁ , a²σ₁² )

\* When Z = aX + bY, the **Linear Combination** of X and Y is given by

Z ~ N (aµ₁ + bµ₂ , a²σ₁² + b²σ₂²)

*Given to find, 2X₁*

Thus, following the property of Multiplication we get

2X₁ ~ N(2µ , 2² σ²)

2X₁ ~ N(2µ , 4 σ²)

And following the property of addition,

X₁ + X₂ ~ N(µ + µ, σ²+σ²) ~ N(2µ , 2σ²)

And the difference between the two is given by

2X₁ - (X₁ + X₂) ~ N(2µ - 2µ₁ , 2σ₁² + 4σ₂²) ~ N(0 , 6σ²)

The mean of 2x₁ and X₁ + X₂ is but the var(σ²) of 2X₁ is 2 times more than the variance of X₁ + X₂.

The difference between the two says that the two given variables are **identically** and **Independently** distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
9. Specify the 5th percentile of profit (in Rupees) for the company
10. Which of the two divisions has a larger probability of making a loss in a given year?